

Super-Enhanced Absorption of Gravitons in Atomic Gases

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Abstract

We present a novel method for detecting gravitons using an atomic gas supported by laser fields. Despite the coupling strength of gravitons to atomic transitions being orders of magnitude weaker than that of photons to atomic transitions, the rate of graviton-absorbed atomic transitions can be substantially elevated to a practically observable level. This enhancement is facilitated by an exceptionally potent amplification effect, stemming from a collective quantum electrodynamics phenomenon that encompasses a simultaneous multiphoton-multiparticle process.

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Relativity and quantum theories emerged in the early twentieth century. Nearly a century later, the direct observation of gravitational waves (GWs) validated a fundamental prediction of general relativity [1–3]. This milestone not only ushered in a new era of astronomical observation but also significantly propelled efforts to unify quantum theory and general relativity. A pivotal question regarding GWs is whether these waves are composed of quantum particles known as gravitons. Experimental verification of gravitons is essential for the unification of quantum and relativity theories. Several theoretical studies have investigated the direct observation of gravitons through ultraweak atomic transitions involving these particles [4–10]. Some proposals for testing the quantum nature of gravity, without detecting gravitons directly, focus on the signals arising from gravity’s quantum properties. These include gravity-induced entanglement between two masses [11, 12], and the fluctuation of the arm length in a GW detector due to quantum states of GWs [13–16]. A recent study [17] by Tobar *et al.* introduces a novel approach for detecting single gravitons in the low-frequency range, highlighting a gravito-phononic analog of the photo-electric effect enabled by advancements in quantum resonators and continuous measurement techniques. In this paper, we propose that ultraweak graviton-absorption atomic transitions can be amplified to observable levels using a recently uncovered quantum enhancement mechanism. This makes the direct observation of gravitons readily realizable in a laboratory setting without necessitating further technological advancements.

The detection of gravitons could, in principle, be achieved through graviton-mediated atomic transitions, analogous to photon-mediated atomic transitions, where, in a simplified approximation, the electron mass acts as the gravitational charge coupling to the quantum gravitational field. Given the extremely weak gravitational coupling—approximately 10^{-43} times weaker than the electromagnetic coupling [18]—it appears challenging to avoid the conclusion that observing gravitons via atomic transitions remains impractical in the near future. However, our recent study shows that an ultraweak atomic transition can be significantly amplified by integrating it with a multiphoton-multiparticle (MPMA) process [19]. This amplification reveals several noteworthy characteristics: it not only delivers substantial enhancements to the transition rate, potentially elevating it by several tens of orders of magnitude, but also exhibits a near-saturation behavior.

In [19], we explored certain ultraweak atomic transitions, such as higher electric multipole E_j transitions ($j = 3, 4, \dots$), in atoms mediated by the absorption of a corresponding

Ej photon, which can be amplified to observable levels through the MPMA process. A general analysis of the strength of these atomic transitions, alongside a comparison with graviton-mediated atomic transitions, offers valuable insights. The Ej transition probability (where $j = 1, 2$ correspond to electric dipole and quadrupole transitions, respectively) scales approximately as $(a/\lambda)^{2j}$, where a denotes the linear size of the atom and λ represents the wavelength of the involved photon [20]. Typically, $(a/\lambda) \approx 10^{-4}$. Consequently, the $E2$ transition probability is on the order of 10^{-8} relative to the $E1$ transition, the $E3$ transition on the order of 10^{-16} , ..., the $E6$ transition on the order of 10^{-48} , and the $E7$ transition on the order of 10^{-56} . The graviton-mediated transition is anticipated to be on the order of $10^{-(8+43)} = 10^{-51}$, where the factor 10^{-8} arises due to that the graviton field is a second-rank tensor, similar to an $E2$ field. For instance, in a hydrogen atom, the $E1$ decay rate (the $2p - 1s$ transition) is on the order of 10^9 s^{-1} . Hence, the graviton-mediated transition rate is projected to be on the order of $10^{-51} \times 10^9 = 10^{-42} \text{ s}^{-1}$, compared to a rate of $5.7 \times 10^{-40} \text{ s}^{-1}$ for the $3d - 1s$ transition, as calculated in [7]. This graviton-mediated transition exhibits a rate relatively close to that of an $E6$ transition but exceeds that of an $E7$ transition. Quantum enhancement through the MPMA process could potentially enable the observation of even higher Ej photoabsorption transitions for $j = 8, 9, \dots$, thereby making it feasible to amplify the graviton-absorption transition to an observable level.

The MPMA process in an atomic gas represents a high-order quantum electrodynamics (QED) phenomenon wherein a specific number of atoms undergo a cooperative transition by simultaneously absorbing laser photons [19, 21–30]. Detailed analyses of the MPMA process and its distinctive properties were presented in our recent studies [19, 30]. Here, we provide a simplified introduction to the process and its enhancement capabilities.

Consider an atomic gas consisting of two species of atoms, designated as A -species and B -species. Within this gas, we examine an m -atom system composed of one A -species atom and $m-1$ B -species atoms. The A -species atom can undergo an $E1$ transition, characterized by an angular transition frequency ω_a between the ground state $|g_a\rangle$ and an excited state $|e_a\rangle$, while each B -species atom possesses an $E1$ transition from the ground state $|g_b\rangle$ to an excited state $|e_b\rangle$, with a transition frequency ω_b .

Two lasers, labeled as \mathfrak{L}_1 and \mathfrak{L}_2 with frequencies $\Omega_{\mathfrak{L}_1}$ and $\Omega_{\mathfrak{L}_2}$, respectively, are utilized to induce atomic transitions. If $\Omega_{\mathfrak{L}_1}$ is tuned near, but not equal to, ω_a , a single-photon absorption process cannot effectively excite an A -species atom. However, the m -atom system

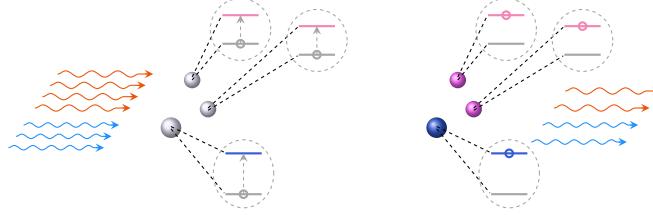


FIG. 1. Schematic plot of a simultaneous three-photon-three-atom process where the atoms are jointly excited. The A -species atom (represented by the large ball) absorbs a laser \mathfrak{L}_1 photon (blue line), while each of two B -species atoms absorbs a laser \mathfrak{L}_2 photon (red line).

can undergo a simultaneous joint transition provided the laser frequencies fulfill the energy conservation condition:

$$\hbar\Omega_{\mathfrak{L}_1} + (m-1)\hbar\Omega_{\mathfrak{L}_2} = \hbar\omega_a + (m-1)\hbar\omega_b, \quad (1)$$

where \hbar represents the reduced Planck constant. In this joint excitation process, the A -species atom absorbs one photon from the laser \mathfrak{L}_1 , while each of the $m-1$ B -species atoms absorbs one photon from the laser \mathfrak{L}_2 (see Fig. 1 for an example with $m=3$).

The transition rate for an m -atom system, denoted as W_{mpma} , is typically low under moderate laser intensity due to its nature as a high-order QED process. However, within this atomic gas, a specific A -species atom, designated as the A_o atom, can form a vast number of m -atom systems with numerous B -species atoms. According to quantum mechanical principles, all these m -atom systems are subject to quantum transitions in parallel, resulting in a significant enhancement of the total transition rate, W_{a_o} , for the A_o atom. Let N_{bo} represent the total number of B -species atoms capable of forming an m -atom system with the A_o atom. The total number of m -atom systems involving the A_o atom, denoted by \mathfrak{N}_{a_o} , is on the order of the combinatorial number: $C_{m-1}^{N_{bo}} = \frac{N_{bo}(N_{bo}-1)\dots(N_{bo}-m+1)}{(m-1)(m-2)\dots 1} \approx N_{bo}^{m-1}/(m-1)!$. Thus, the total transition rate can be expressed as:

$$W_{a_o} = \mathfrak{N}_{a_o} W_{mpma} \approx \frac{N_{bo}^{m-1}}{(m-1)!} W_{mpma}. \quad (2)$$

For instance, with $N_{bo} \approx 10^{12}$ and $m=8$, \mathfrak{N}_{a_o} can reach values as high as 10^{91} in principle, signifying an exceptional enhancement factor. This yields a considerable total transition rate for the A_o atom.

This enhancement can be understood intuitively: in science and technology, it is frequently observed that while the signal from a single sample may be faint, a sufficiently

large number of samples can generate a significant total signal. In typical atomic transition events, the number of samples corresponds to the number of atoms in the system, with the practical limit generally being below Avogadro's number, which limits the maximum possible enhancement. However, in this multiatom process, the number of "samples" is determined by a combinatorial factor rather than merely the number of atoms, enabling the total number to become exceptionally large.

In a homogeneous atomic gas, the value of N_{b0} is determined by $N_{b0} \approx \rho_b l_{mpma}^3$, where ρ_b represents the atomic density of B -species atoms and l_{mpma} is a fundamental length which defines the maximum linear size of the m -atom system enabling the simultaneous MPMA transition [19, 30]. The length l_{mpma} is primarily governed by the uncertainty principle in quantum mechanics and can be approximated as $l_{mpma} = ac/(\Omega_{\mathfrak{L}_1} - \omega_a)$, where c is the speed of light and α is a constant smaller than but on the order of unity.

Further analysis reveals that this enhancement mechanism incorporates a regulatory near-saturation effect for W_{a_0} [19]. This implies that, while W_{a_0} is a rapidly increasing function of both N_{b0} and m in its bare form, its maximum possible value remains below the scale of approximately 10^9 s $^{-1}$. This regulation is facilitated by the self-tuning of l_{mpma} , which adjusts to reduce the values of N_{b0} and, consequently, W_{a_0} , thereby ensuring compliance with relativistic causality [19].

It is instructive to estimate W_{mpma} , which can be calculated using perturbation theory and approximated as follows [19]:

$$W_{mpma} \approx C \Gamma_a n_{\mathfrak{L}_1} \Omega_{\mathfrak{L}_1} \frac{\Gamma^2 n_{\mathfrak{L}_2}^{m-1} \tau_b^{m-1} \Omega_{\mathfrak{L}_2}^{m-1}}{(\Omega_{\mathfrak{L}_2} - \omega_b)^{2m}} [f(m)]^{2m} \rho(E_f) \Big|_{E_f = \varepsilon_e^a + (m-1)\varepsilon_e^b}. \quad (3)$$

In this expression, $C = (\Omega_{\mathfrak{L}_1}/\omega_a)(\Omega_{\mathfrak{L}_2}/\omega_b)^{m-1}/(2^{4m+1}\pi^{3m-1}\hbar^2) \approx 1/(2^{4m+1}\pi^{3m-1}\hbar^2)$; $\Gamma_a = 4\alpha_e \hbar \omega_a^3 |\langle e_a | \mathbf{d} | g_a \rangle|^2 / 3e^2 c^2$ denotes an energy width parameter associated with the $E1$ transition of the A -species atom (with α_e being the fine-structure constant, e the elementary charge, and \mathbf{d} the electric dipole moment operator); $\tau_b = 4\alpha_e \omega_b^3 |\langle e_b | \mathbf{d} | g_b \rangle|^2 / 3e^2 c^2$; and $\Gamma \approx \hbar \tau_b$ represents an approximately averaged energy width parameter [19, 30]. The term $n_{\mathfrak{L}_i}$ ($i = 1, 2$) represents the number of laser photons in laser \mathfrak{L}_i within a volume of $\lambda_i^3 = (2\pi c)^3 / \Omega_{\mathfrak{L}_i}^3$. The function $f(m)$ takes values approximately in the range $(1/m, 1)$, and $\rho(E_f)$ denotes the density of states of the m -atom system at energy $E_f = \varepsilon_e^a + (m-1)\varepsilon_e^b$, with ε_e^a and ε_e^b representing the eigenenergies of states $|e_a\rangle$ and $|e_b\rangle$, respectively [19, 30].

By combining Equations (2) and (3) and performing some algebraic manipulation, one

can express W_{a_o} in the following approximate form:

$$W_{a_o} \approx \frac{1}{32\pi^2\hbar^2} n_{\mathfrak{L}_1} \Gamma_a \Omega_{\mathfrak{L}_1} \frac{\Gamma^2}{(\Omega_{\mathfrak{L}_2} - \omega_b)^2} \mathfrak{E}_{mpma}^{m-1} \rho(E_f). \quad (4)$$

Here $\mathfrak{E}_{mpma}^{m-1}$ denotes a composite enhancement factor, given by:

$$\mathfrak{E}_{mpma}^{m-1} = \left[\frac{n_{\mathfrak{L}_2} \tau_b \Omega_{\mathfrak{L}_2}}{16\pi^3 (\Omega_{\mathfrak{L}_2} - \omega_b)^2} \frac{[f(m)]^{\frac{2m}{m-1}}}{[(m-1)!]^{\frac{1}{m-1}}} N_{b_0} \right]^{m-1}. \quad (5)$$

For typical atomic species, τ_b is on the order of $2\pi \times 10$ MHz, while ω_b is on the order of $2\pi \times 5 \times 10^{14}$ Hz. The detuning parameter $|\Omega_{\mathfrak{L}_2} - \omega_b|$ can be set to around $2\pi \times 10^{10}$ Hz practically. The enhancement factor $\mathfrak{E}_{mpma}^{m-1}$ can be on the order of, or greater than, $(0.1 n_{\mathfrak{L}_2} N_{b_0}/m)^{m-1}$. Even with a very small value of $n_{\mathfrak{L}_2}$, such as 10^{-4} , which corresponds to a weak laser intensity, a sufficiently large N_{b_0} , on the order of 10^{12} or greater, enables the enhancement factor $\mathfrak{E}_{mpma}^{m-1}$ to reach an exceptionally large value when m is substantial.

This MPMA process can be incorporated into an ultraweak atomic transition, such as the atomic absorption of an $E5$ -photon, to enhance its transition rate [19]. Consider a similar m -atom system, where the atomic transitions of the B -species atoms remain the same, but the transition of the A -species atom is replaced by the ultraweak $E5$ -photon atomic transition. In this case, the role of the laser \mathfrak{L}_1 is substituted by a flux of $E5$ photons. The frequencies of the involved photons must satisfy the condition of overall energy conservation for the joint process, which is expressed as:

$$\hbar\Omega_{E_5} + (m-1)\hbar\Omega_{\mathfrak{L}_2} = \hbar\omega_{a,E_5} + (m-1)\hbar\omega_b, \quad (6)$$

where Ω_{E_5} denotes the frequency of the $E5$ photons and ω_{a,E_5} denotes the transition frequency for the $E5$ transition of the A -species atom. The transition rate for a specific A_o atom in an atomic gas can be approximated as analogous to Eq. (3):

$$W_{a_o,E_5} \approx \frac{1}{32\pi^2\hbar^2} n_{E_5} \Gamma_{a,n_{E_5}} \Omega_{E_5} \frac{\Gamma^2}{(\Omega_{\mathfrak{L}_2} - \omega_b)^2} \mathfrak{E}_{mpma}^{m-1} \rho(E_f). \quad (7)$$

In this expression, n_{E_5} denotes the number of $E5$ photons in a volume of $\lambda_{E_5}^3 = (2\pi c)^3/\Omega_{E_5}^3$. Γ_{a,E_5} is an energy width parameter associated with the ultraweak $E5$ transition of the A -species atom, $\Gamma_{a,E_5} \sim \alpha_e \hbar\omega_{a,E_5} |\langle e_{a,E_5} | r^5 Y_5 | g_a \rangle|^2 / \lambda_{E_5}^{10}$ (r is the radius of the electron, Y is the spherical harmonic function, and $|e_{a,E_5}\rangle$ is the corresponding excited state), which could be 10^{-40} smaller than the width parameter Γ_a for the $E1$ transition. However, the enhancement

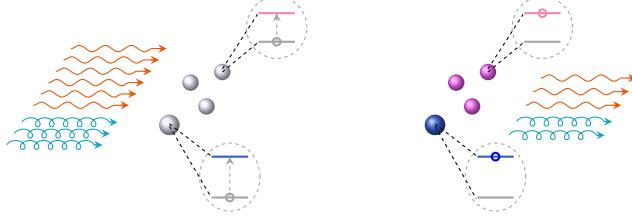


FIG. 2. Schematic illustration of a graviton-absorption atomic process involving simultaneous joint excitations of a four-atom system. The A -species atom (represented by the large ball) absorbs a graviton (curly line), while each of three B -species atoms absorbs a laser photon (wavy line).

factor $\mathfrak{E}_{mpma}^{m-1}$ can be made to sufficiently large to overcome the smallness of Γ_{a,E_5} , allowing the transition rate W_{a_o,E_5} to reach a detectable level.

We can extend this MPMA enhancement to atomic transitions involving graviton absorption, in a manner analogous to the ultraweak atomic transition involving an $E5$ photon. Consider a flux of gravitons with frequency Ω_{gr} . A suitable A -species atom and $m - 1$ B -species atoms can be selected, along with a laser of frequency $\Omega_{\mathfrak{L}_2}$, such that the following energy conservation condition is satisfied:

$$\hbar\Omega_{gr} + (m - 1)\hbar\Omega_{\mathfrak{L}_2} = \hbar\omega_{a,gr} + (m - 1)\hbar\omega_b, \quad (8)$$

where $\omega_{a,gr}$ denotes the transition frequency for the graviton-absorptive transition of the A -species atom. In this scenario, an analogous MPMA process of this m -atom system can occur. The A -species atom absorbs the graviton while, simultaneously, the $m - 1$ B -species atoms undergo transitions by each absorbing a laser photon (see Fig. 2 for an example with $m = 4$).

Now, consider an atomic gas undergoing graviton absorption. The overall transition rate of a specific A -species atom in the gas can be analyzed in a manner similar to previous treatments and can be approximated in a form analogous to Eq. (3):

$$W_{a_o,gr} \approx \frac{1}{32\pi^2\hbar^2} n_{gr} \Gamma_{a,gr} \Omega_{gr} \frac{\Gamma^2}{(\Omega_{\mathfrak{L}_2} - \omega_b)^2} \mathfrak{E}_{mpma}^{m-1} \rho(E_f). \quad (9)$$

Here, n_{gr} denotes the number of gravitons in a volume of $\lambda_{gr}^3 = (2\pi c)^3/\Omega_{gr}^3$, and $\Gamma_{a,gr}$ represents an energy width parameter associated with the graviton-absorption transition of the A -species atom. Formally, the interaction operator (density) between the atom and a

weak gravitational field can be approximated as [7, 8]:

$$H_{ge} \approx \frac{m_e}{2} R_{0i0j}(t, \mathbf{x}) x^i x^j. \quad (10)$$

In this expression, m_e is the electron mass and x^i denotes the Fermi normal coordinate in the atom's rest space, with i, j as spatial indices. The term R refers to the curvature tensor operator of the gravitational field. The energy width parameter $\Gamma_{a,gr} \sim (G m_e^2 \omega_{a,gr} / c) |\langle e_{a,gr} | x^i x^j | g_a \rangle|^2 / \lambda_{gr}^4$, where G is the gravitational constant and $|e_{a,gr}\rangle$ is the corresponding excited state, is roughly 10^{-50} smaller than width parameter Γ_a for the $E1$ transition. However, owing to the potentially enormous value of $\mathfrak{E}_{mpma}^{m-1}$, the transition rate $W_{a_o,gr}$ can reach detectable level despite the extremely small magnitude of $\Gamma_{a,gr}$.

In the graviton-absorption process, the incoming flux of natural gravitons may sometimes be extremely low—potentially below one graviton per minute per square centimeter. In such cases, the primary interest could be the absorption probability of a graviton. If the frequency of an incoming graviton satisfies the energy condition (Eq. (8)), enabling the joint MPMA process to occur, the absorption probability is significantly enhanced. Although the probability of a single m -atom system absorbing an incoming graviton remains exceedingly small, the existence of a vast number of such systems, all capable of absorbing gravitons in parallel, allows the total absorption probability for the entire sample to approach unity.

A graviton carries an intrinsic angular momentum of $2\hbar$, which generally requires that for an A -species atom to absorb a graviton, the corresponding atomic transition must involve a change of two units of angular momentum—analogous to an $E2$ photoabsorption transition. Once excited, the A -species atom can often emit an $E1$ photon by transitioning from the excited state to a distinct lower-energy excited state, rather than the ground state. This $E1$ photon, distinguished by its characteristic frequency, acts as the observable signature of graviton absorption.

An alternative class of material systems exists in which graviton absorption occurs via quantum transitions between states that are not eigenstates of the angular momentum operator. Consequently, the conventional selection rules tied to angular momentum quantum numbers do not apply. These systems consist of ion-doped crystals, where the doped ions—such as rare-earth elements like Eu^{3+} , Nd^{3+} , and Pr^{3+} —serve as active sites for graviton-involved multiphoton-multiparticle transitions, also referred to as the (generalized) MPMA process. Compared to atomic gases, ion-doped crystals can be more beneficial for graviton

absorption. In these solid-state systems, the doped ions can easily achieve high densities, reaching 10^{18} ions/cm $^{-3}$ even at very low doping rates. To implement MPMA-assisted graviton absorption in the crystal, two quantum transitions within the same ion species can be selected, eliminating the need for two distinct species. For instance, the transition between the ground state and the first excited state can be used for laser photon absorption, mirroring the transitions of *B*-species atoms in atomic gases. Meanwhile, another transition—between the ground state and a second excited state, well-separated in energy from the first excited state—can facilitate graviton absorption, analogous to the transitions in *A*-species atoms in atomic gases.

An important factor concerning MPMA process in ion-doped crystals is the inhomogeneous broadening of the ions' optical transitions, denoted by Γ_{inh} . In some systems, Γ_{inh}/\hbar can be as large as 10^2 GHz or more; however, it can be reduced to several hundred MHz in certain systems [31–34] with low-doping rates. Another interesting property of rare-earth ion optical transitions is their narrow natural linewidths, Γ_{ion}/\hbar , which are typically on the order of 10 kHz or even smaller. Due to inhomogeneous broadening, not all possible m -ion systems within a length scale l_{mpma} contribute equally, as their excitation energies exhibit a spread of approximately $m^\gamma \Gamma_{inh}$, where the exponent γ ranges between 0.5 and 1. As a result, only a fraction of these m -ion systems—roughly $\Gamma_{ion}/m^\gamma \Gamma_{inh}$ or larger—effectively participates in the MPMA process, introducing a reduction factor into the transition rate. However, this reduction is readily offset by the vastly larger combinatorial number of possible m -ion systems, and the graviton absorption process can still reach a near-saturation regime. In this regime, l_{mpma} self-adjusts to constrain what would otherwise be an unphysically large transition rate. Another relevant factor is the thermal broadening of the excited states of the ions, and the crystal can be cooled to cryogenic temperatures of a few kelvins to reduce the thermal broadening width.

For graviton absorption to occur, a corresponding graviton source is required. The possibility of enhanced graviton emission in atomic gases or ion-doped crystals is explored in a separate study. If such enhancement is realized, gravitons could be generated with well-controlled frequencies, offering a more tunable approach to graviton production.

Natural high-frequency graviton sources include solar gravitons [9, 35, 36] and relic gravitons. The estimated power of solar graviton emission is approximately 10^5 W in the optical frequency range [36], corresponding to a flux of about 10 gravitons per square centimeter per

day at Earth’s solar distance. Given the broad frequency distribution of solar gravitons, enhancing the detection bandwidth for graviton absorption becomes an essential consideration. In ion-doped crystals, the detection bandwidth depends on the spectral broadening of the excited levels involved in the MPMA process. Denoting the total broadening—including both inhomogeneous and thermal broadening—of an ion’s excited level as Γ_{tot} , the total excitation energy of an m -ion system exhibits a spectral spread of approximately $m^\gamma \Gamma_{tot}$, which defines the detection bandwidth. A larger Γ_{tot} thus increases the detection bandwidth, though it may simultaneously reduce the number of m -ion systems capable of absorbing gravitons, thereby influencing the MPMA process. Nevertheless, as previously argued, this reduction can be counterbalanced, and the MPMA process can still reach a near-saturation regime. Additionally, under the same laser field, an ion-doped crystal can respond to solar gravitons across different frequency regimes by adapting different m values. Consequently, considering multiple m values, the total detection bandwidth could reach 10³ GHz or beyond, making the detection of solar gravitons more feasible.

It is interesting to explore the possibility of extending graviton absorption into the infrared and ultraviolet frequency ranges. Expanding detection into the ultraviolet range appears relatively straightforward and can be achieved using methods analogous to those employed in the optical frequency range. However, extending detection into the far-infrared range may present challenges and requires further investigation. Broadening the detectable frequency range in this manner could significantly expand the observational window for studying astronomical gravitons, opening new opportunities for exploration and discovery.

In summary, we propose that atomic graviton absorption can be readily observed with the assistance of an MPMA process. Detecting this quantum phenomenon is not only essential for understanding the nature of gravitational waves but also fundamental to advancing our knowledge of the quantum framework itself.

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